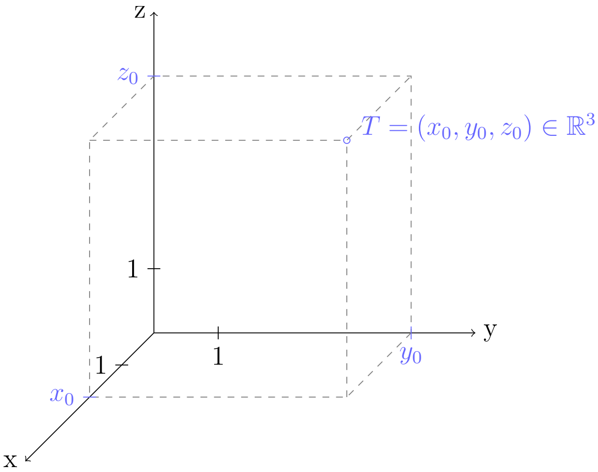
6.1. Geometrijski vektori u prostoru

15. 1. 2020.

Fotografija: Ajay Tripathi

Kartezijev koordinatni sustav u prostoru

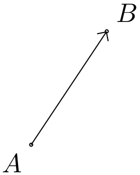


Posredstvom Kartezijeva koordinatnog sustava točke u prostoru identificiramo s elementima skupa

$$\mathbb{R}^3 := \{(x, y, z) : x, y, z \in \mathbb{R}\}.$$

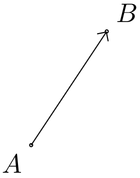
Geometrijski vektori u prostoru

Točke $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3) \in \mathbb{R}^3$ određuju **(geometrijski) vektor** $\vec{v}$:= skup koji čine orijentirana dužina $\overrightarrow{AB}$



i sve orijentirane dužine u prostoru koje se iz nje dobiju translacijom (tzv. **reprezentanti vektora** $\vec{v}$).

Točke $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3) \in \mathbb{R}^3$ određuju **(geometrijski) vektor** $\vec{v}$:= skup koji čine orijentirana dužina $\overrightarrow{AB}$

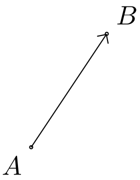


i sve orijentirane dužine u prostoru koje se iz nje dobiju translacijom (tzv. **reprezentanti vektora** $\vec{v}$).
Neprecizno, ali uobičajeno i korisno, pišemo

$$\vec{v} = \overrightarrow{AB}.$$

Geometrijski vektori u prostoru

Točke $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3) \in \mathbb{R}^3$ određuju **(geometrijski) vektor** $\vec{v}$:= skup koji čine orijentirana dužina $\overrightarrow{AB}$



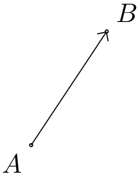
i sve orijentirane dužine u prostoru koje se iz nje dobiju translacijom (tzv. **reprezentanti vektora** $\vec{v}$).
Neprecizno, ali uobičajeno i korisno, pišemo

$$\vec{v} = \overrightarrow{AB}.$$

Skup svih vektora u prostoru $=: V^3$.

Geometrijski vektori u prostoru

Točke $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3) \in \mathbb{R}^3$ određuju **(geometrijski) vektor** $\vec{v}$:= skup koji čine orijentirana dužina $\overrightarrow{AB}$



i sve orijentirane dužine u prostoru koje se iz nje dobiju translacijom (tzv. **reprezentanti vektora** $\vec{v}$).
Neprecizno, ali uobičajeno i korisno, pišemo

$$\vec{v} = \overrightarrow{AB}.$$

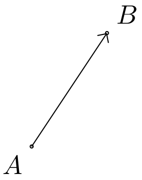
Skup svih vektora u prostoru $=: V^3$.

Poseban slučaj: ako je $A = B$, tada je $\overrightarrow{AB} = \overrightarrow{AA} =: \vec{0}$ (tzv. **nulvektor**):

Analitička geometrija povezuje geometriju i algebru

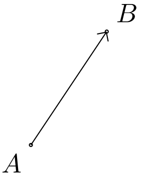
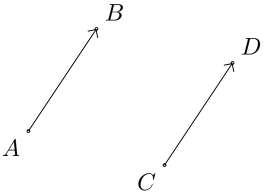
GEOMETRIJA	$\leftrightarrow$	ALGEBRA

Analitička geometrija povezuje geometriju i algebru

GEOMETRIJA	$\leftrightarrow$	ALGEBRA
	$\leftrightarrow$	$\overrightarrow{AB} = [b_1 - a_1, b_2 - a_2, b_3 - a_3]$

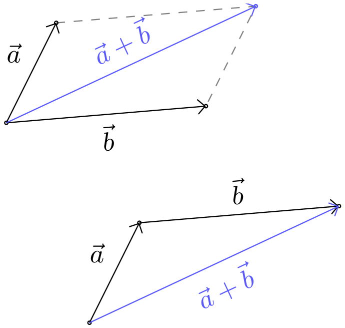
(Ovdje su $A = (a_1, a_2, a_3), B = (b_1, b_2, b_3) \in \mathbb{R}^3$.)

Analitička geometrija povezuje geometriju i algebru

GEOMETRIJA	$\leftrightarrow$	ALGEBRA
	$\leftrightarrow$	$\overrightarrow{AB} = [b_1 - a_1, b_2 - a_2, b_3 - a_3]$
 Ista duljina, smjer i orijentacija.	$\overrightarrow{AB} = \overrightarrow{CD}$ $\leftrightarrow$	$[b_1 - a_1, b_2 - a_2, b_3 - a_3]$ $\parallel$ $[d_1 - c_1, d_2 - c_2, d_3 - c_3]$ Isti koordinatni zapisi.

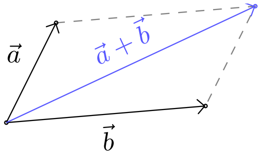
(Ovdje su $A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3)$, $C = (c_1, c_2, c_3)$, $D = (d_1, d_2, d_3) \in \mathbb{R}^3$.)

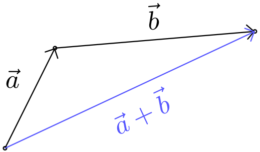
Zbrajanje vektora u V^3

GEOMETRIJA	$\leftrightarrow$	ALGEBRA
	$\leftrightarrow$	$[a_1, a_2, a_3] + [b_1, b_2, b_3]$ $= [a_1 + b_1, a_2 + b_2, a_3 + b_3]$

Zbrajanje vektora u V^3

GEOMETRIJA $\leftrightarrow$ ALGEBRA





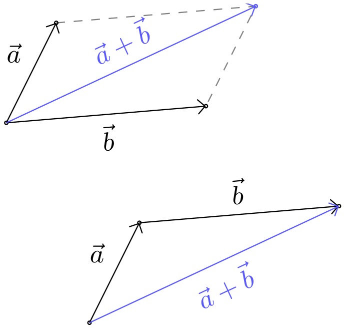
$\leftrightarrow$

$$[a_1, a_2, a_3] + [b_1, b_2, b_3]$$
$$= [a_1 + b_1, a_2 + b_2, a_3 + b_3]$$

PR.: $[1, 2, 3] + [4, 5, 6] =$

Zbrajanje vektora u V^3

GEOMETRIJA	$\leftrightarrow$	ALGEBRA

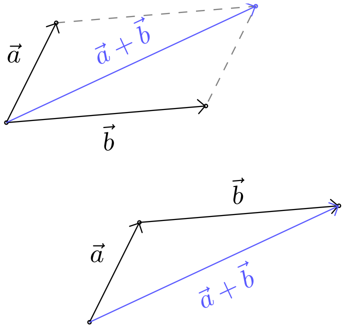


$\leftrightarrow$

$$[a_1, a_2, a_3] + [b_1, b_2, b_3]$$
$$= [a_1 + b_1, a_2 + b_2, a_3 + b_3]$$

PR.: $[1, 2, 3] + [4, 5, 6] = [1 + 4, 2 + 5, 3 + 6]$

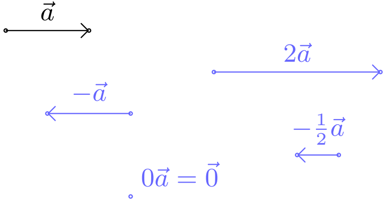
Zbrajanje vektora u V^3

GEOMETRIJA	$\leftrightarrow$	ALGEBRA
	$\leftrightarrow$	$[a_1, a_2, a_3] + [b_1, b_2, b_3] = [a_1 + b_1, a_2 + b_2, a_3 + b_3]$

PR.: $[1, 2, 3] + [4, 5, 6] = [1 + 4, 2 + 5, 3 + 6] = [5, 7, 9]$.

Množenje skalarom $\alpha \in \mathbb{R}$ u V^3

GEOMETRIJA $\leftrightarrow$ ALGEBRA

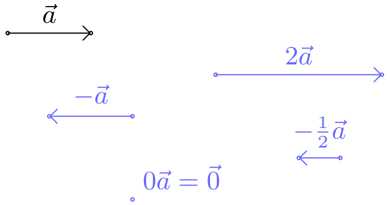


$\leftrightarrow$

$$\alpha\,[a_1, a_2, a_3] = [\alpha a_1, \alpha a_2, \alpha a_3]$$

Množenje skalarom $\alpha \in \mathbb{R}$ u V^3

GEOMETRIJA $\leftrightarrow$ ALGEBRA



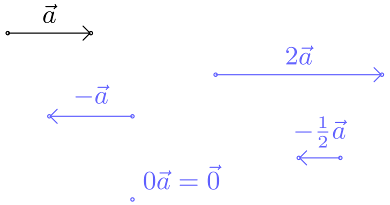
$\leftrightarrow$

$$\alpha\,[a_1, a_2, a_3] = [\alpha a_1, \alpha a_2, \alpha a_3]$$

PR.: $-\frac{1}{2}\left[\frac{1}{2}, 2, -3\right] =$

Množenje skalarom $\alpha \in \mathbb{R}$ u V^3

GEOMETRIJA $\leftrightarrow$ ALGEBRA



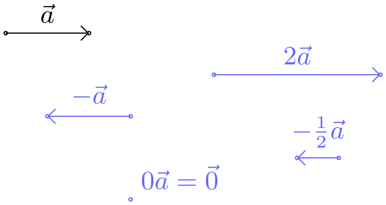
$\leftrightarrow$

$$\alpha\,[a_1, a_2, a_3] = [\alpha a_1, \alpha a_2, \alpha a_3]$$

PR.: $-\frac{1}{2}\left[\frac{1}{2}, 2, -3\right] = \left[-\frac{1}{2}\cdot\frac{1}{2}, -\frac{1}{2}\cdot 2, -\frac{1}{2}\cdot(-3)\right]$

Množenje skalarom $\alpha \in \mathbb{R}$ u V^3

GEOMETRIJA $\leftrightarrow$ ALGEBRA



$\leftrightarrow$

$$\alpha\,[a_1, a_2, a_3] = [\alpha a_1, \alpha a_2, \alpha a_3]$$

PR.: $-\frac{1}{2}\left[\frac{1}{2}, 2, -3\right] = \left[-\frac{1}{2}\cdot\frac{1}{2}, -\frac{1}{2}\cdot 2, -\frac{1}{2}\cdot(-3)\right] = \left[-\frac{1}{4}, -1, \frac{3}{2}\right].$

Kolinearni vektori

Neka su $\vec{a}, \vec{b} \in V^3$. Ako postoji $\alpha \in \mathbb{R}$ takav da je

$$\vec{b} = \alpha \vec{a} \quad \text{ili} \quad \vec{a} = \alpha \vec{b},$$

kažemo da su $\vec{a}$ i $\vec{b}$ **kolinearni** i pišemo

$$\vec{a} \parallel \vec{b}.$$

(Geometrijski: $\vec{a}$ i $\vec{b}$ su kolinearni $\Leftrightarrow$ kad ih nanesemo iz iste točke, leže na istom pravcu.)

Kolinearni vektori

Neka su $\vec{a}, \vec{b} \in V^3$. Ako postoji $\alpha \in \mathbb{R}$ takav da je

$$\vec{b} = \alpha\vec{a} \quad \text{ili} \quad \vec{a} = \alpha\vec{b},$$

kažemo da su $\vec{a}$ i $\vec{b}$ **kolinearni** i pišemo

$$\vec{a} \parallel \vec{b}.$$

(Geometrijski: $\vec{a}$ i $\vec{b}$ su kolinearni $\Leftrightarrow$ kad ih nanesemo iz iste točke, leže na istom pravcu.)

PR.:

- $[2, 1, 1] \parallel [-4, -2, -2]$.

Kolinearni vektori

Neka su $\vec{a}, \vec{b} \in V^3$. Ako postoji $\alpha \in \mathbb{R}$ takav da je

$$\vec{b} = \alpha \vec{a} \quad \text{ili} \quad \vec{a} = \alpha \vec{b},$$

kažemo da su $\vec{a}$ i $\vec{b}$ **kolinearni** i pišemo

$$\vec{a} \parallel \vec{b}.$$

(Geometrijski: $\vec{a}$ i $\vec{b}$ su kolinearni $\Leftrightarrow$ kad ih nanesemo iz iste točke, leže na istom pravcu.)

PR.:

- $[2, 1, 1] \parallel [-4, -2, -2]$.
- $[1, 0, 0]$ i $[0, 1, 1]$ nisu kolinearni.

Kolinearni vektori

Neka su $\vec{a}, \vec{b} \in V^3$. Ako postoji $\alpha \in \mathbb{R}$ takav da je

$$\vec{b} = \alpha \vec{a} \quad \text{ili} \quad \vec{a} = \alpha \vec{b},$$

kažemo da su $\vec{a}$ i $\vec{b}$ **kolinearni** i pišemo

$$\vec{a} \parallel \vec{b}.$$

(Geometrijski: $\vec{a}$ i $\vec{b}$ su kolinearni $\Leftrightarrow$ kad ih nanesemo iz iste točke, leže na istom pravcu.)

PR.:

- $[2, 1, 1] \parallel [-4, -2, -2]$.
- $[1, 0, 0]$ i $[0, 1, 1]$ nisu kolinearni.
- $\vec{0}$ je kolinearan sa svim vektorima.

Baza prostora V^3

Skup sastavljen od triju $\overbrace{\text{nekomplanarnih}}^{\text{kad ih nanesemo iz iste točke, ne leže svi u istoj ravnini}}$ vektora iz V^3 zovemo **bazom prostora** V^3.

Baza prostora V^3

Skup sastavljen od triju $\overbrace{\text{nekomplanarnih}}^{\text{kad ih nanesemo iz iste točke, ne leže svi u istoj ravnini}}$ vektora iz V^3 zovemo **bazom prostora** V^3.

Osnovno svojstvo baze

Neka je $\left\{\vec{b}_1, \vec{b}_2, \vec{b}_3\right\}$ baza prostora V^3. Tada svaki $\vec{v} \in V^3$ ima jedinstven prikaz

$$\vec{v} = \alpha_1 \vec{b}_1 + \alpha_2 \vec{b}_2 + \alpha_3 \vec{b}_3$$

za neke $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$. Taj prikaz zovemo **koordinatnim zapisom** od $\vec{v}$ u bazi $\left\{\vec{b}_1, \vec{b}_2, \vec{b}_3\right\}$.
Koeficijente $\alpha_1, \alpha_2, \alpha_3$ zovemo **koordinatama** vektora $\vec{v}$ u bazi $\left\{\vec{b}_1, \vec{b}_2, \vec{b}_3\right\}$.

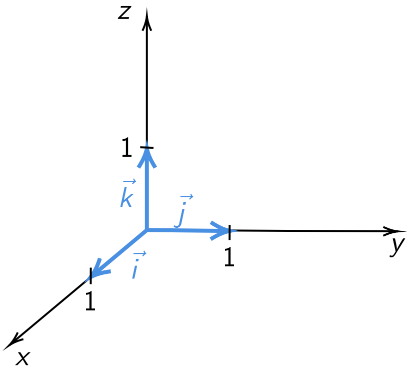
Primjer 1

Standardna/kanonska baza za V^3 je

$$\left\{\vec{i}, \vec{j}, \vec{k}\right\},$$

gdje je

$$\vec{i} := [1,0,0],\ \vec{j} := [0,1,0],\ \vec{k} := [0,0,1].$$



Za svaki $\vec{a} = [a_1, a_2, a_3] \in V^3$ vrijedi $\boxed{\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}.}$

Još četiri pojma za lagan život u V^3

1. Skalarni produkt u V^3
2. Duljina vektora u V^3
3. Vektorski produkt u V^3
4. Mješoviti produkt u V^3

1) Skalarni produkt u V^3

Definiramo **skalarni produkt** $\vec{a} \cdot \vec{b}$ vektora $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ formulom

$$\begin{aligned} \vec{a} \cdot \vec{b} &:= a_1 b_1 + a_2 b_2 + a_3 b_3 \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos \measuredangle(\vec{a}, \vec{b}), \end{aligned}$$

gdje su $|\vec{a}|$ i $|\vec{b}|$ duljine vektora $\vec{a}$ odnosno $\vec{b}$.

1) Skalarni produkt u V^3

Definiramo **skalarni produkt** $\vec{a} \cdot \vec{b}$ vektora $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ formulom

$$\begin{aligned} \vec{a} \cdot \vec{b} &:= a_1 b_1 + a_2 b_2 + a_3 b_3 \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos \measuredangle(\vec{a}, \vec{b}), \end{aligned}$$

gdje su $|\vec{a}|$ i $|\vec{b}|$ duljine vektora $\vec{a}$ odnosno $\vec{b}$.

Primijetimo: $\vec{a} \cdot \vec{b} \in \mathbb{R}$.

1) Skalarni produkt u V^3

Definiramo **skalarni produkt** $\vec{a} \cdot \vec{b}$ vektora $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ formulom

$$\begin{aligned} \vec{a} \cdot \vec{b} &:= a_1 b_1 + a_2 b_2 + a_3 b_3 \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos \measuredangle(\vec{a}, \vec{b}), \end{aligned}$$

gdje su $|\vec{a}|$ i $|\vec{b}|$ duljine vektora $\vec{a}$ odnosno $\vec{b}$.

Primijetimo: $\vec{a} \cdot \vec{b} \in \mathbb{R}$.

Vrijedi:

$$\vec{a} \cdot \vec{b} = 0 \quad \Leftrightarrow \quad \vec{a} \perp \vec{b}.$$

1) Skalarni produkt u V^3

Definiramo **skalarni produkt** $\vec{a} \cdot \vec{b}$ vektora $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ formulom

$$\begin{aligned}\vec{a} \cdot \vec{b} &:= a_1 b_1 + a_2 b_2 + a_3 b_3 \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos \measuredangle(\vec{a}, \vec{b}),\end{aligned}$$

gdje su $|\vec{a}|$ i $|\vec{b}|$ duljine vektora $\vec{a}$ odnosno $\vec{b}$.

Primijetimo: $\vec{a} \cdot \vec{b} \in \mathbb{R}$.

Vrijedi:

$$\vec{a} \cdot \vec{b} = 0 \quad \Leftrightarrow \quad \vec{a} \perp \vec{b}.$$

PR.: Imamo

$$[1, 2, 3] \cdot [4, -5, 2] =$$

1) Skalarni produkt u V^3

Definiramo **skalarni produkt** $\vec{a} \cdot \vec{b}$ vektora $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ formulom

$$\begin{aligned} \vec{a} \cdot \vec{b} &:= a_1 b_1 + a_2 b_2 + a_3 b_3 \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos \measuredangle(\vec{a}, \vec{b}), \end{aligned}$$

gdje su $|\vec{a}|$ i $|\vec{b}|$ duljine vektora $\vec{a}$ odnosno $\vec{b}$.

Primijetimo: $\vec{a} \cdot \vec{b} \in \mathbb{R}$.

Vrijedi:

$$\vec{a} \cdot \vec{b} = 0 \quad \Leftrightarrow \quad \vec{a} \perp \vec{b}.$$

PR.: Imamo

$$[1, 2, 3] \cdot [4, -5, 2] = 1 \cdot 4 + 2 \cdot (-5) + 3 \cdot 2$$

1) Skalarni produkt u V^3

Definiramo **skalarni produkt** $\vec{a} \cdot \vec{b}$ vektora $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ formulom

$$\begin{aligned} \vec{a} \cdot \vec{b} &:= a_1 b_1 + a_2 b_2 + a_3 b_3 \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos \measuredangle(\vec{a}, \vec{b}), \end{aligned}$$

gdje su $|\vec{a}|$ i $|\vec{b}|$ duljine vektora $\vec{a}$ odnosno $\vec{b}$.

Primijetimo: $\vec{a} \cdot \vec{b} \in \mathbb{R}$.

Vrijedi:

$$\vec{a} \cdot \vec{b} = 0 \quad \Leftrightarrow \quad \vec{a} \perp \vec{b}.$$

PR.: Imamo

$$[1, 2, 3] \cdot [4, -5, 2] = 1 \cdot 4 + 2 \cdot (-5) + 3 \cdot 2 = 0,$$

1) Skalarni produkt u V^3

Definiramo **skalarni produkt** $\vec{a} \cdot \vec{b}$ vektora $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ formulom

$$\begin{aligned} \vec{a} \cdot \vec{b} &:= a_1 b_1 + a_2 b_2 + a_3 b_3 \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos \measuredangle(\vec{a}, \vec{b}), \end{aligned}$$

gdje su $|\vec{a}|$ i $|\vec{b}|$ duljine vektora $\vec{a}$ odnosno $\vec{b}$.

Primijetimo: $\vec{a} \cdot \vec{b} \in \mathbb{R}$.

Vrijedi:

$$\vec{a} \cdot \vec{b} = 0 \quad \Leftrightarrow \quad \vec{a} \perp \vec{b}.$$

PR.: Imamo

$$[1, 2, 3] \cdot [4, -5, 2] = 1 \cdot 4 + 2 \cdot (-5) + 3 \cdot 2 = 0,$$

dakle

$$[1, 2, 3] \perp [4, -5, 2].$$

Svojstva skalarnog produkta u V^3

Za sve $\vec{u}, \vec{v}, \vec{w} \in V^3$ i $\alpha \in \mathbb{R}$ vrijedi:

- $\vec{u} \cdot \vec{u} \geq 0$
- $\vec{u} \cdot \vec{u} = 0 \; \Leftrightarrow \; \vec{u} = \vec{0}$
- $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$
- $(\alpha v) \cdot \vec{w} = \alpha(\vec{v} \cdot \vec{w})$.

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| =$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2}$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $|\vec{j}|$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $|\vec{j}| = |[0, 1, 0]|$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $|\vec{j}| = |[0, 1, 0]| = \sqrt{0^2 + 1^2 + 0^2}$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $\left|\vec{j}\right| = |[0, 1, 0]| = \sqrt{0^2 + 1^2 + 0^2} = 1$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $|\vec{j}| = |[0, 1, 0]| = \sqrt{0^2 + 1^2 + 0^2} = 1$
- $\vec{i} \cdot \vec{k} =$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $\left|\vec{j}\right| = |[0, 1, 0]| = \sqrt{0^2 + 1^2 + 0^2} = 1$
- $\vec{i} \cdot \vec{k} = [1, 0, 0] \cdot [0, 0, 1]$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $|\vec{j}| = |[0, 1, 0]| = \sqrt{0^2 + 1^2 + 0^2} = 1$
- $\vec{i} \cdot \vec{k} = [1, 0, 0] \cdot [0, 0, 1] = 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $|\vec{j}| = |[0, 1, 0]| = \sqrt{0^2 + 1^2 + 0^2} = 1$
- $\vec{i} \cdot \vec{k} = [1, 0, 0] \cdot [0, 0, 1] = 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 = 0$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}$, $\vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $|\vec{j}| = |[0, 1, 0]| = \sqrt{0^2 + 1^2 + 0^2} = 1$
- $\vec{i} \cdot \vec{k} = [1, 0, 0] \cdot [0, 0, 1] = 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 = 0 \;\Rightarrow\; \vec{i} \perp \vec{k}.$

2) Duljina vektora u V^3

Duljina $|\vec{a}|$ vektora $\vec{a} = [a_1, a_2, a_3] \in V^3$ dana je formulom

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{\vec{a} \cdot \vec{a}}.$$

PR.: $|[1, -2, 0]| = \sqrt{1^2 + (-2)^2 + 0^2} = \sqrt{5}.$

PR.: Vektori $\vec{i}, \vec{j}$ i $\vec{k}$ su duljine 1 i međusobno okomiti, npr.:

- $|\vec{j}| = |[0, 1, 0]| = \sqrt{0^2 + 1^2 + 0^2} = 1$
- $\vec{i} \cdot \vec{k} = [1, 0, 0] \cdot [0, 0, 1] = 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 = 0 \;\Rightarrow\; \vec{i} \perp \vec{k}.$

$\rightsquigarrow$ Kažemo da je baza $\left\{\vec{i}, \vec{j}, \vec{k}\right\}$ **ortonormirana**.

Zadatak 55(a)

Zadana je baza $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su i vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $|\vec{v}|$ i $|\vec{w}|$.

Zadatak 55(a)

Zadana je baza $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su i vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $|\vec{v}|$ i $|\vec{w}|$.

Rješenje. Imamo

$$|\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}}$$

Zadatak 55(a)

Zadana je baza $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su i vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $|\vec{v}|$ i $|\vec{w}|$.

Rješenje. Imamo

$$\begin{aligned} |\vec{v}| &= \sqrt{\vec{v} \cdot \vec{v}} \\ &= \sqrt{(\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{a} - \vec{b} + \vec{c})} \end{aligned}$$

Zadatak 55(a)

Zadana je baza $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su i vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $|\vec{v}|$ i $|\vec{w}|$.

Rješenje. Imamo

$$\begin{aligned}
|\vec{v}| &= \sqrt{\vec{v} \cdot \vec{v}} \\
&= \sqrt{(\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{a} - \vec{b} + \vec{c})} \\
&= \sqrt{\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} - 2\vec{a} \cdot \vec{b} + 2\vec{a} \cdot \vec{c} - 2\vec{b} \cdot \vec{c}}
\end{aligned}$$

Zadatak 55(a)

Zadana je baza $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su i vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $|\vec{v}|$ i $|\vec{w}|$.

Rješenje. Imamo

$$\begin{aligned}
|\vec{v}| &= \sqrt{\vec{v} \cdot \vec{v}} \\
&= \sqrt{(\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{a} - \vec{b} + \vec{c})} \\
&= \sqrt{\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} - 2\vec{a} \cdot \vec{b} + 2\vec{a} \cdot \vec{c} - 2\vec{b} \cdot \vec{c}} \\
&= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 - 2|\vec{a}| \cdot |\vec{b}| \cdot \cos\gamma + 2|\vec{a}| \cdot |\vec{c}| \cdot \cos\beta - 2|\vec{b}| \cdot |\vec{c}| \cdot \cos\alpha}
\end{aligned}$$

Zadatak 55(a)

Zadana je baza $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su i vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $|\vec{v}|$ i $|\vec{w}|$.

Rješenje. Imamo

$$\begin{aligned}
|\vec{v}| &= \sqrt{\vec{v} \cdot \vec{v}} \\
&= \sqrt{(\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{a} - \vec{b} + \vec{c})} \\
&= \sqrt{\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} - 2\vec{a} \cdot \vec{b} + 2\vec{a} \cdot \vec{c} - 2\vec{b} \cdot \vec{c}} \\
&= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 - 2|\vec{a}| \cdot |\vec{b}| \cdot \cos\gamma + 2|\vec{a}| \cdot |\vec{c}| \cdot \cos\beta - 2|\vec{b}| \cdot |\vec{c}| \cdot \cos\alpha} \\
&= \sqrt{1^2 + 2^2 + 3^2 - 2 \cdot 1 \cdot 2 \cdot \tfrac{1}{2} + 2 \cdot 1 \cdot 3 \cdot \tfrac{1}{2} - 2 \cdot 2 \cdot 3 \cdot 0} = \sqrt{15}
\end{aligned}$$

Zadatak 55(a)

Zadana je baza $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su i vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $|\vec{v}|$ i $|\vec{w}|$.

Rješenje. Imamo

$$\begin{aligned}
|\vec{v}| &= \sqrt{\vec{v} \cdot \vec{v}} \\
&= \sqrt{(\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{a} - \vec{b} + \vec{c})} \\
&= \sqrt{\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} - 2\vec{a} \cdot \vec{b} + 2\vec{a} \cdot \vec{c} - 2\vec{b} \cdot \vec{c}} \\
&= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 - 2|\vec{a}| \cdot |\vec{b}| \cdot \cos\gamma + 2|\vec{a}| \cdot |\vec{c}| \cdot \cos\beta - 2|\vec{b}| \cdot |\vec{c}| \cdot \cos\alpha} \\
&= \sqrt{1^2 + 2^2 + 3^2 - 2 \cdot 1 \cdot 2 \cdot \tfrac{1}{2} + 2 \cdot 1 \cdot 3 \cdot \tfrac{1}{2} - 2 \cdot 2 \cdot 3 \cdot 0} = \sqrt{15}
\end{aligned}$$

i analogno $|\vec{w}| \overset{\text{sami}}{=} \sqrt{13}$.

Zadatak 55(b)

Neka je $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle\left(\vec{b}, \vec{c}\right) = \frac{\pi}{2}, \quad \beta := \measuredangle\left(\vec{a}, \vec{c}\right) = \frac{\pi}{3}, \quad \gamma := \measuredangle\left(\vec{a}, \vec{b}\right) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle\left(\vec{v}, \vec{w}\right)$.

Zadatak 55(b)

Neka je $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos \measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|}$$

Zadatak 55(b)

Neka je $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos\measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \overset{\text{Zad. 55(a)}}{=} \frac{\vec{v} \cdot \vec{w}}{\sqrt{15} \cdot \sqrt{13}}$$

Zadatak 55(b)

Neka je $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos \measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \overset{\text{Zad. 55(a)}}{=} \frac{\vec{v} \cdot \vec{w}}{\sqrt{15} \cdot \sqrt{13}}$$

$$\vec{v} \cdot \vec{w} = (\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c}$$

Zadatak 55(b)

Neka je $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos \measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \overset{\text{Zad. 55(a)}}{=} \frac{\vec{v} \cdot \vec{w}}{\sqrt{15} \cdot \sqrt{13}}$$

$$\begin{aligned} \vec{v} \cdot \vec{w} &= (\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos\gamma + |\vec{a}| \cdot |\vec{c}| \cdot \cos\beta - |\vec{b}|^2 + |\vec{c}|^2 \end{aligned}$$

Zadatak 55(b)

Neka je $\{\vec{a}, \vec{b}, \vec{c}\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos \measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \overset{\text{Zad. 55(a)}}{=} \frac{\vec{v} \cdot \vec{w}}{\sqrt{15} \cdot \sqrt{13}}$$

$$\begin{aligned}
\vec{v} \cdot \vec{w} &= (\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} \\
&= |\vec{a}| \cdot |\vec{b}| \cdot \cos\gamma + |\vec{a}| \cdot |\vec{c}| \cdot \cos\beta - |\vec{b}|^2 + |\vec{c}|^2 \\
&= 1 \cdot 2 \cdot \frac{1}{2} + 1 \cdot 3 \cdot \frac{1}{2} - 2^2 + 3^2
\end{aligned}$$

Zadatak 55(b)

Neka je $\{\vec{a}, \vec{b}, \vec{c}\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos \measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \overset{\text{Zad. 55(a)}}{=} \frac{\vec{v} \cdot \vec{w}}{\sqrt{15} \cdot \sqrt{13}}$$

$$\begin{aligned}
\vec{v} \cdot \vec{w} &= (\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} \\
&= |\vec{a}| \cdot |\vec{b}| \cdot \cos \gamma + |\vec{a}| \cdot |\vec{c}| \cdot \cos \beta - |\vec{b}|^2 + |\vec{c}|^2 \\
&= 1 \cdot 2 \cdot \frac{1}{2} + 1 \cdot 3 \cdot \frac{1}{2} - 2^2 + 3^2 = \frac{15}{2}.
\end{aligned}$$

Zadatak 55(b)

Neka je $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos \measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \overset{\text{Zad. 55(a)}}{=} \frac{\vec{v} \cdot \vec{w}}{\sqrt{15} \cdot \sqrt{13}} = \frac{\frac{15}{2}}{\sqrt{15} \cdot \sqrt{13}}$$

$$\begin{aligned} \vec{v} \cdot \vec{w} &= (\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos\gamma + |\vec{a}| \cdot |\vec{c}| \cdot \cos\beta - |\vec{b}|^2 + |\vec{c}|^2 \\ &= 1 \cdot 2 \cdot \frac{1}{2} + 1 \cdot 3 \cdot \frac{1}{2} - 2^2 + 3^2 = \frac{15}{2}. \end{aligned}$$

Zadatak 55(b)

Neka je $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos\measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \overset{\text{Zad. 55(a)}}{=} \frac{\vec{v} \cdot \vec{w}}{\sqrt{15} \cdot \sqrt{13}} = \frac{\frac{15}{2}}{\sqrt{15} \cdot \sqrt{13}} = \frac{1}{2}\sqrt{\frac{15}{13}},$$

pri čemu predzadnja jednakost vrijedi jer je

$$\begin{aligned} \vec{v} \cdot \vec{w} &= (\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} \\ &= |\vec{a}| \cdot |\vec{b}| \cdot \cos\gamma + |\vec{a}| \cdot |\vec{c}| \cdot \cos\beta - |\vec{b}|^2 + |\vec{c}|^2 \\ &= 1 \cdot 2 \cdot \frac{1}{2} + 1 \cdot 3 \cdot \frac{1}{2} - 2^2 + 3^2 = \frac{15}{2}. \end{aligned}$$

Zadatak 55(b)

Neka je $\left\{\vec{a}, \vec{b}, \vec{c}\right\}$ baza prostora V^3 takva da je

$$|\vec{a}| = 1, \quad |\vec{b}| = 2, \quad |\vec{c}| = 3, \quad \alpha := \measuredangle(\vec{b}, \vec{c}) = \frac{\pi}{2}, \quad \beta := \measuredangle(\vec{a}, \vec{c}) = \frac{\pi}{3}, \quad \gamma := \measuredangle(\vec{a}, \vec{b}) = \frac{\pi}{3}.$$

Zadani su vektori $\vec{v} := \vec{a} - \vec{b} + \vec{c}$ i $\vec{w} := \vec{b} + \vec{c}$. Izračunajte $\measuredangle(\vec{v}, \vec{w})$.

Rješenje. $\measuredangle(\vec{v}, \vec{w}) \in [0, \pi]$ zadovoljava

$$\cos \measuredangle(\vec{v}, \vec{w}) = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} \overset{\text{Zad. 55(a)}}{=} \frac{\vec{v} \cdot \vec{w}}{\sqrt{15} \cdot \sqrt{13}} = \frac{\frac{15}{2}}{\sqrt{15} \cdot \sqrt{13}} = \frac{1}{2}\sqrt{\frac{15}{13}},$$

pri čemu predzadnja jednakost vrijedi jer je

$$\begin{aligned}
\vec{v} \cdot \vec{w} &= (\vec{a} - \vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{c} \\
&= |\vec{a}| \cdot |\vec{b}| \cdot \cos\gamma + |\vec{a}| \cdot |\vec{c}| \cdot \cos\beta - |\vec{b}|^2 + |\vec{c}|^2 \\
&= 1 \cdot 2 \cdot \frac{1}{2} + 1 \cdot 3 \cdot \frac{1}{2} - 2^2 + 3^2 = \frac{15}{2}.
\end{aligned}$$

Dakle, $\measuredangle(\vec{v}, \vec{w}) = \arccos\left(\frac{1}{2}\sqrt{\frac{15}{13}}\right)$.

3) Vektorski produkt u V^3

Za $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ definiramo

$$\vec{a} \times \vec{b} := [a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1]$$

3) Vektorski produkt u V^3

Za $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ definiramo

$$\begin{aligned}\vec{a} \times \vec{b} &:= [a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1] \\ &= \underbrace{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}_{\substack{\text{zapis pomoću} \\ \text{determinante}}}.\end{aligned}$$

3) Vektorski produkt u V^3

Za $\vec{a} = [a_1, a_2, a_3], \vec{b} = [b_1, b_2, b_3] \in V^3$ definiramo

$$\begin{aligned}\vec{a} \times \vec{b} &:= [a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1] \\ &= \underbrace{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}_{\substack{\text{zapis pomoću} \\ \text{determinante}}}.\end{aligned}$$

Primijetimo: $\vec{a} \times \vec{b} \in V^3$.

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} :=$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} =$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3$

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \qquad\qquad\qquad\qquad} \qquad (*)$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i}} \qquad (*)$$

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j}} \qquad (*)$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}.} \qquad (*)$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}.} \qquad (*)$$

Dokaz jednakosti $(*)$.

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo
$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$
PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

- Vrijedi
$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}.} \qquad (*)$$

Dokaz jednakosti $(*)$. Imamo

$$\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}.} \qquad (*)$$

Dokaz jednakosti $(*)$. Imamo

$$\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$

$$= (a_2 b_3 - a_3 b_2)[1, 0, 0] - (a_1 b_3 - a_3 b_1)[0, 1, 0] + (a_1 b_2 - a_2 b_1)[0, 0, 1]$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

- Za $a, b, c, d \in \mathbb{R}$ definiramo

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} := ad - bc.$$

PR.: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}.} \qquad (*)$$

Dokaz jednakosti $(*)$. Imamo

$$\begin{aligned} &\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k} \\ &= (a_2 b_3 - a_3 b_2)[1,0,0] - (a_1 b_3 - a_3 b_1)[0,1,0] + (a_1 b_2 - a_2 b_1)[0,0,1] \\ &= [a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1]. \end{aligned}$$

Primjer

Imamo

$$[1, 1, 3] \times [2, 1, -1] =$$

Primjer

Imamo

$$[1,1,3]\times[2,1,-1]=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 3\\ 2 & 1 & -1\end{vmatrix}$$

Imamo

$$[1,1,3] \times [2,1,-1] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} \vec{i}$$

Primjer

Imamo

$$[1,1,3] \times [2,1,-1] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} \vec{j}$$

Imamo

$$[1,1,3]\times[2,1,-1]=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 3\\ 2 & 1 & -1\end{vmatrix}$$

$$=\begin{vmatrix}1 & 3\\ 1 & -1\end{vmatrix}\vec{i}-\begin{vmatrix}1 & 3\\ 2 & -1\end{vmatrix}\vec{j}+\begin{vmatrix}1 & 1\\ 2 & 1\end{vmatrix}\vec{k}$$

Primjer

Imamo

$$
\begin{aligned}
[1,1,3] \times [2,1,-1] &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \vec{k} \\
&= (1 \cdot (-1) - 3 \cdot 1)\vec{i} - (1 \cdot (-1) - 3 \cdot 2)\vec{j} + (1 \cdot 1 - 1 \cdot 2)\vec{k}
\end{aligned}
$$

Primjer

Imamo

$$
\begin{aligned}
[1, 1, 3] \times [2, 1, -1] &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \vec{k} \\
&= (1 \cdot (-1) - 3 \cdot 1)\vec{i} - (1 \cdot (-1) - 3 \cdot 2)\vec{j} + (1 \cdot 1 - 1 \cdot 2)\vec{k} \\
&= -4\vec{i} + 7\vec{j} - \vec{k}
\end{aligned}
$$

Primjer

Imamo

$$
\begin{aligned}
[1,1,3]\times[2,1,-1] &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix} \\
&= \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}\vec{k} \\
&= (1\cdot(-1) - 3\cdot 1)\vec{i} - (1\cdot(-1) - 3\cdot 2)\vec{j} + (1\cdot 1 - 1\cdot 2)\vec{k} \\
&= -4\vec{i} + 7\vec{j} - \vec{k} \\
&= [-4, 7, -1].
\end{aligned}
$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{matrix}}$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

- Vrijedi

$$+$$

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{matrix} = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k}}$$

- Vrijedi

$$+$$

$$\left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \end{array}\right. = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k} - b_1 a_2 \vec{k} - b_2 a_3 \vec{i} - b_3 a_1 \vec{j}.$$

$$-$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

- Vrijedi

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{matrix} = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k} - b_1 a_2 \vec{k} - b_2 a_3 \vec{i} - b_3 a_1 \vec{j}.}$$

Primjer. Imamo

$$[1, 1, 3] \times [2, 1, -1] =$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

- Vrijedi

$$\begin{array}{|ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \end{array} = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k} - b_1 a_2 \vec{k} - b_2 a_3 \vec{i} - b_3 a_1 \vec{j}.$$

Primjer. Imamo

$$[1,1,3] \times [2,1,-1] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix}$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

- Vrijedi

$$+$$

$$\left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \end{array}\right. = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k} - b_1 a_2 \vec{k} - b_2 a_3 \vec{i} - b_3 a_1 \vec{j}.$$

$$-$$

Primjer. Imamo

$$[1,1,3] \times [2,1,-1] = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & 1 & 3 & 1 & 1 \\ 2 & 1 & -1 & 2 & 1 \end{array}\right.$$

- Vrijedi

$$+$$

$$\boxed{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{matrix} = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k} - b_1 a_2 \vec{k} - b_2 a_3 \vec{i} - b_3 a_1 \vec{j}.}$$

$$-$$

Primjer. Imamo

$$[1,1,3] \times [2,1,-1] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 1 & 1 \\ 2 & 1 \end{matrix}$$

$$= -\vec{i} + 6\vec{j} + \vec{k}$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

- Vrijedi

$$\left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \end{array}\right. = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k} - b_1 a_2 \vec{k} - b_2 a_3 \vec{i} - b_3 a_1 \vec{j}.$$

Primjer. Imamo

$$[1,1,3] \times [2,1,-1] = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & 1 & 3 & 1 & 1 \\ 2 & 1 & -1 & 2 & 1 \end{array}\right.$$

$$= -\vec{i} + 6\vec{j} + \vec{k} - 2\vec{k} - 3\vec{i} + \vec{j}$$

- Vrijedi

$$+$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ a_1 & a_2 \\ b_1 & b_2 \end{matrix} = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k} - b_1 a_2 \vec{k} - b_2 a_3 \vec{i} - b_3 a_1 \vec{j}.$$

$$-$$

Primjer. Imamo

$$[1,1,3] \times [2,1,-1] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 1 & 1 \\ 2 & 1 \end{matrix}$$

$$= -\vec{i} + 6\vec{j} + \vec{k} - 2\vec{k} - 3\vec{i} + \vec{j}$$

$$= -4\vec{i} + 7\vec{j} - \vec{k}$$

- Vrijedi

$$\boxed{\left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ a_1 & a_2 & a_3 & a_1 & a_2 \\ b_1 & b_2 & b_3 & b_1 & b_2 \end{array}\right. = a_2 b_3 \vec{i} + a_3 b_1 \vec{j} + a_1 b_2 \vec{k} - b_1 a_2 \vec{k} - b_2 a_3 \vec{i} - b_3 a_1 \vec{j}.}$$

Primjer. Imamo

$$\begin{aligned} [1,1,3] \times [2,1,-1] &= \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & 1 & 3 & 1 & 1 \\ 2 & 1 & -1 & 2 & 1 \end{array}\right. \\ &= -\vec{i} + 6\vec{j} + \vec{k} - 2\vec{k} - 3\vec{i} + \vec{j} \\ &= -4\vec{i} + 7\vec{j} - \vec{k} \\ &= [-4, 7, -1]. \end{aligned}$$

Svojstva vektorskog produkta u V^3

Neka su $\vec{a}, \vec{b} \in V^3$. Vrijedi:

- $\vec{a} \times \vec{b} \perp \vec{a}, \vec{b}$.

Svojstva vektorskog produkta u V^3

Neka su $\vec{a}, \vec{b} \in V^3$. Vrijedi:

- $\vec{a} \times \vec{b} \perp \vec{a}, \vec{b}$.
- $\vec{a} \times \vec{b} = \vec{0} \iff \vec{a} \parallel \vec{b}$.

Svojstva vektorskog produkta u V^3

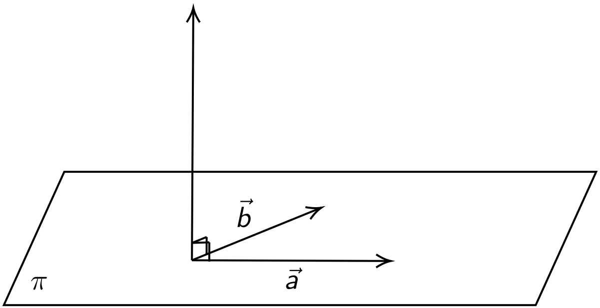
Neka su $\vec{a}, \vec{b} \in V^3$. Vrijedi:

- $\vec{a} \times \vec{b} \perp \vec{a}, \vec{b}$.
- $\vec{a} \times \vec{b} = \vec{0} \;\Leftrightarrow\; \vec{a} \parallel \vec{b}$.
- $\left|\vec{a} \times \vec{b}\right| =$ površina paralelograma razapetog vektorima $\vec{a}$ i $\vec{b}$.

Svojstva vektorskog produkta u V^3

Neka su $\vec{a}, \vec{b} \in V^3$. Vrijedi:

- $\vec{a} \times \vec{b} \perp \vec{a}, \vec{b}$.
- $\vec{a} \times \vec{b} = \vec{0} \;\Leftrightarrow\; \vec{a} \parallel \vec{b}$.
- $\left|\vec{a} \times \vec{b}\right| =$ površina paralelograma razapetog vektorima $\vec{a}$ i $\vec{b}$.

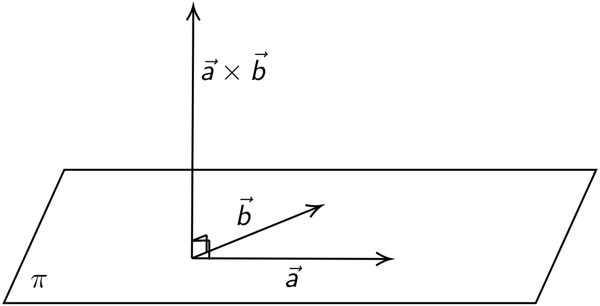


Jako važna napomena. Ako $\vec{a} \nparallel \vec{b}$ i trebamo vektor $\vec{v} \neq \vec{0}$ takav da je $\vec{v} \perp \vec{a}, \vec{b}$,

Svojstva vektorskog produkta u V^3

Neka su $\vec{a}, \vec{b} \in V^3$. Vrijedi:

- $\vec{a} \times \vec{b} \perp \vec{a}, \vec{b}$.
- $\vec{a} \times \vec{b} = \vec{0} \iff \vec{a} \parallel \vec{b}$.
- $\left|\vec{a} \times \vec{b}\right|$ = površina paralelograma razapetog vektorima $\vec{a}$ i $\vec{b}$.



Jako važna napomena. Ako $\vec{a} \nparallel \vec{b}$ i trebamo vektor $\vec{v} \neq \vec{0}$ takav da je $\vec{v} \perp \vec{a}, \vec{b}$, možemo uzeti

$$\vec{v} = \vec{a} \times \vec{b}.$$

Mješoviti produkt u V^3

Mješoviti produkt vektora $\vec{a}, \vec{b}, \vec{c} \in V^3$ definira se formulom

$$\left(\vec{a}, \vec{b}, \vec{c}\right) := \vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$$

Mješoviti produkt u V^3

Mješoviti produkt vektora $\vec{a}, \vec{b}, \vec{c} \in V^3$ definira se formulom

$$(\vec{a}, \vec{b}, \vec{c}\,) := \vec{a} \cdot (\vec{b} \times \vec{c}) = \underbrace{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}_{\text{determinanta reda 3}} \in \mathbb{R}.$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

računa se Laplaceovim razvojem po prvom retku (kao kod vektorskog produkta) ili, općenitije, Laplaceovim razvojem po proizvoljnom retku ili stupcu, prema sljedećoj tablici predznaka:

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

računa se Laplaceovim razvojem po prvom retku (kao kod vektorskog produkta) ili, općenitije, Laplaceovim razvojem po proizvoljnom retku ili stupcu, prema sljedećoj tablici predznaka:

$$\begin{array}{c|c|c} + & - & + \\ - & + & - \\ + & - & + \end{array}$$

Npr. razvojem po drugom stupcu dobivamo

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} =$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

računa se Laplaceovim razvojem po prvom retku (kao kod vektorskog produkta) ili, općenitije, Laplaceovim razvojem po proizvoljnom retku ili stupcu, prema sljedećoj tablici predznaka:

$$\begin{matrix} + & \boxed{\begin{matrix} - \\ + \\ - \end{matrix}} & + \\ - & & - \\ + & & + \end{matrix}$$

Npr. razvojem po drugom stupcu dobivamo

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -\,a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

računa se Laplaceovim razvojem po prvom retku (kao kod vektorskog produkta) ili, općenitije, Laplaceovim razvojem po proizvoljnom retku ili stupcu, prema sljedećoj tablici predznaka:

$$\begin{matrix} + & \boxed{-} & + \\ - & + & - \\ + & - & + \end{matrix}$$

Npr. razvojem po drugom stupcu dobivamo

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + b_2 \begin{vmatrix} a_1 & a_3 \\ c_1 & c_3 \end{vmatrix}$$

Objašnjenje zapisa pomoću determinante – 1. način: Laplaceov razvoj

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

računa se Laplaceovim razvojem po prvom retku (kao kod vektorskog produkta) ili, općenitije, Laplaceovim razvojem po proizvoljnom retku ili stupcu, prema sljedećoj tablici predznaka:

$$\begin{array}{c|c|c} + & - & + \\ - & + & - \\ + & - & + \end{array}$$

Npr. razvojem po drugom stupcu dobivamo

$$\begin{vmatrix} a_1 & \boxed{a_2} & a_3 \\ b_1 & \boxed{b_2} & b_3 \\ c_1 & \boxed{c_2} & c_3 \end{vmatrix} = -a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + b_2 \begin{vmatrix} a_1 & a_3 \\ c_1 & c_3 \end{vmatrix} - c_2 \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}.$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

može se izračunati i Sarrusjevim pravilom:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

može se izračunati i Sarrusjevim pravilom:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \begin{matrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{matrix}$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

može se izračunati i Sarrusjevim pravilom:

$$\overset{+}{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}\begin{matrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{matrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2$$

Objašnjenje zapisa pomoću determinante – 2. način: Sarrusjevo pravilo

Determinanta

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

može se izračunati i Sarrusjevim pravilom:

$$\overset{+}{}$$

$$\left|\begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix}\right|\begin{matrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{matrix} = a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 - c_1 b_2 a_3 - c_2 b_3 a_1 - c_3 b_1 a_2 .$$

$$-$$

Svojstva mješovitog produkta u V^3

Za sve $\vec{a}, \vec{b}, \vec{c} \in V^3$ vrijedi:

- $(\vec{a}, \vec{b}, \vec{c}) = (\vec{b}, \vec{c}, \vec{a}) = (\vec{c}, \vec{a}, \vec{b})$.

Svojstva mješovitog produkta u V^3

Za sve $\vec{a}, \vec{b}, \vec{c} \in V^3$ vrijedi:

- $(\vec{a}, \vec{b}, \vec{c}) = (\vec{b}, \vec{c}, \vec{a}) = (\vec{c}, \vec{a}, \vec{b}) = -(\vec{b}, \vec{a}, \vec{c}) = -(\vec{a}, \vec{c}, \vec{b}) = -(\vec{c}, \vec{b}, \vec{a})$.

Svojstva mješovitog produkta u V^3

Za sve $\vec{a}, \vec{b}, \vec{c} \in V^3$ vrijedi:

- $(\vec{a}, \vec{b}, \vec{c}) = (\vec{b}, \vec{c}, \vec{a}) = (\vec{c}, \vec{a}, \vec{b}) = -(\vec{b}, \vec{a}, \vec{c}) = -(\vec{a}, \vec{c}, \vec{b}) = -(\vec{c}, \vec{b}, \vec{a})$.
- $|(\vec{a}, \vec{b}, \vec{c})| =$ volumen paralelepipeda razapetog vektorima $\vec{a}$, $\vec{b}$ i $\vec{c}$.

Svojstva mješovitog produkta u V^3

Za sve $\vec{a}, \vec{b}, \vec{c} \in V^3$ vrijedi:

- $(\vec{a}, \vec{b}, \vec{c}) = (\vec{b}, \vec{c}, \vec{a}) = (\vec{c}, \vec{a}, \vec{b}) = -(\vec{b}, \vec{a}, \vec{c}) = -(\vec{a}, \vec{c}, \vec{b}) = -(\vec{c}, \vec{b}, \vec{a})$.
- $|(\vec{a}, \vec{b}, \vec{c})| =$ volumen paralelepipeda razapetog vektorima $\vec{a}$, $\vec{b}$ i $\vec{c}$.
- $(\vec{a}, \vec{b}, \vec{c}) = 0 \;\Leftrightarrow\; \vec{a}, \vec{b}$ i $\vec{c}$ su komplanarni.

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1, 2, 3], \ \ \vec{v}_2 := [-1, 4, 9] \text{ i } \vec{v}_3 := [1, 0, -1]$$

komplanarni.

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1, 2, 3], \;\; \vec{v}_2 := [-1, 4, 9] \text{ i } \vec{v}_3 := [1, 0, -1]$$

komplanarni.

Rješenje. Budući da je

$$\left(\vec{v}_1, \vec{v}_2, \vec{v}_3\right) = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 4 & 9 \\ 1 & 0 & -1 \end{vmatrix}$$

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1, 2, 3], \;\; \vec{v}_2 := [-1, 4, 9] \text{ i } \vec{v}_3 := [1, 0, -1]$$

komplanarni.

Rješenje. Budući da je

$$(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \begin{vmatrix} 1 & \boxed{\begin{matrix} 2 \\ 4 \\ 0 \end{matrix}} & 3 \\ -1 & & 9 \\ 1 & & -1 \end{vmatrix}$$

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1, 2, 3],\ \ \vec{v}_2 := [-1, 4, 9] \text{ i } \vec{v}_3 := [1, 0, -1]$$

komplanarni.

Rješenje. Budući da je

$$\begin{aligned}
\left(\vec{v}_1, \vec{v}_2, \vec{v}_3\right) &= \begin{vmatrix} 1 & \boxed{\begin{matrix} 2 \\ 4 \\ 0 \end{matrix}} & 3 \\ -1 & & 9 \\ 1 & & -1 \end{vmatrix} \\
&= -2 \cdot \begin{vmatrix} -1 & 9 \\ 1 & -1 \end{vmatrix}
\end{aligned}$$

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1, 2, 3], \ \vec{v}_2 := [-1, 4, 9] \text{ i } \vec{v}_3 := [1, 0, -1]$$

komplanarni.

Rješenje. Budući da je

$$\begin{aligned}
\left(\vec{v}_1, \vec{v}_2, \vec{v}_3\right) &= \begin{vmatrix} 1 & \boxed{\begin{matrix} 2 \\ 4 \\ 0 \end{matrix}} & \begin{matrix} 3 \\ 9 \\ -1 \end{matrix} \end{vmatrix} \\
&= -2 \cdot \begin{vmatrix} -1 & 9 \\ 1 & -1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix}
\end{aligned}$$

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1, 2, 3], \;\; \vec{v}_2 := [-1, 4, 9] \text{ i } \vec{v}_3 := [1, 0, -1]$$

komplanarni.

Rješenje. Budući da je

$$\begin{aligned}
(\vec{v}_1, \vec{v}_2, \vec{v}_3) &= \begin{vmatrix} 1 & \boxed{\begin{matrix} 2 \\ 4 \\ 0 \end{matrix}} & \begin{matrix} 3 \\ 9 \\ -1 \end{matrix} \end{vmatrix} \\
&= -2 \cdot \begin{vmatrix} -1 & 9 \\ 1 & -1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 3 \\ -1 & 9 \end{vmatrix}
\end{aligned}$$

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1,2,3],\ \ \vec{v}_2 := [-1,4,9] \text{ i } \vec{v}_3 := [1,0,-1]$$

komplanarni.

Rješenje. Budući da je

$$\begin{aligned}
(\vec{v}_1, \vec{v}_2, \vec{v}_3) &= \begin{vmatrix} 1 & 2 & 3 \\ -1 & 4 & 9 \\ 1 & 0 & -1 \end{vmatrix} \\
&= -2 \cdot \begin{vmatrix} -1 & 9 \\ 1 & -1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 3 \\ -1 & 9 \end{vmatrix} \\
&= -2\,(-1 \cdot (-1) - 9 \cdot 1) + 4\,(1 \cdot (-1) - 3 \cdot 1) - 0
\end{aligned}$$

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1, 2, 3], \;\; \vec{v}_2 := [-1, 4, 9] \text{ i } \vec{v}_3 := [1, 0, -1]$$

komplanarni.

Rješenje. Budući da je

$$\begin{aligned}
\left(\vec{v}_1, \vec{v}_2, \vec{v}_3\right) &= \begin{vmatrix} 1 & \boxed{\begin{matrix} 2 \\ 4 \\ 0 \end{matrix}} & 3 \\ -1 & & 9 \\ 1 & & -1 \end{vmatrix} \\
&= -2 \cdot \begin{vmatrix} -1 & 9 \\ 1 & -1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 3 \\ -1 & 9 \end{vmatrix} \\
&= -2\left(-1 \cdot (-1) - 9 \cdot 1\right) + 4\left(1 \cdot (-1) - 3 \cdot 1\right) - 0 \\
&= 0,
\end{aligned}$$

Zadatak 56

Ispitajte jesu li vektori

$$\vec{v}_1 := [1, 2, 3],\ \ \vec{v}_2 := [-1, 4, 9] \text{ i } \vec{v}_3 := [1, 0, -1]$$

komplanarni.

Rješenje. Budući da je

$$\begin{aligned}
\left(\vec{v}_1, \vec{v}_2, \vec{v}_3\right) &= \begin{vmatrix} 1 & \boxed{\begin{matrix} 2 \\ 4 \\ 0 \end{matrix}} & 3 \\ -1 & & 9 \\ 1 & & -1 \end{vmatrix} \\
&= -2 \cdot \begin{vmatrix} -1 & 9 \\ 1 & -1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 3 \\ -1 & 9 \end{vmatrix} \\
&= -2\,(-1 \cdot (-1) - 9 \cdot 1) + 4\,(1 \cdot (-1) - 3 \cdot 1) - 0 \\
&= 0,
\end{aligned}$$

vektori $\vec{v}_1$, $\vec{v}_2$ i $\vec{v}_3$ su komplanarni.